Matematikk 1

Denne fila inneholder løsningsforslag til følgende ekstra øvingsoppgaver til kapittel 7.1 – 7.2:

7.1.1 a - c
7.1.4 a - f
7.1.5 a og b
7.1.7 a - f
7.1.8 a - d
7.1.9 a - c
7.1.10 a - c
7.1.11 a - c
7.1.12 a og b
7.1.13 a
7.1.14 a og b
7.1.15 a og b

Delkapittel 7.1

oppgave 1

a) $f(x) = 3x+5$

$$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)+5-(3x+5)}{h}$$

$$= \frac{\cancel{3x}+3h+\cancel{5}-\cancel{3x}-\cancel{5}}{h} = \frac{3\cancel{h}}{\cancel{h}} = \underline{3}$$

$$f'(x) = \underline{3}$$

b) $f(x) = 2x^2 - 4x + 3$

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2 - 4(x+h)+3-(2x^2-4x+3)}{h}$$

$$= \frac{\cancel{2x^2}+4hx+2h^2-\cancel{4x}-4h+\cancel{3}-\cancel{2x^2}+\cancel{4x}-\cancel{3}}{h}$$

$$= \frac{4hx+2h^2-4h}{h}$$

$$= \frac{\cancel{h}(2h+4x-4)}{\cancel{h}} = \underline{2h+4x-4}$$

$$\boxed{\begin{array}{l}\text{Lar:} \\ \lim h \to 0 \text{ og får:} \\ f'(x) = \underline{4x-4}\end{array}}$$

c) $f(x) = (3x+2)(4x-3)$

$$\frac{f(x+h)-f(x)}{h} = \frac{(3(x+h)+2)(4(x+h)-3)-(3x+2)(4x-3)}{h}$$

$$= \frac{(3x+3h+2)(4x+4h-3)-(3x+2)(4x-3)}{h}$$

Se tabell 7.2 for derivasjonsregler!

$$= \frac{\cancel{12x^2}+12hx-\cancel{9x}+12hx+12h^2-9h+\cancel{8x}+8h-\cancel{6}-(\cancel{12x^2}-\cancel{9x}+\cancel{8x}-\cancel{6})}{h}$$

$$= \frac{12h^2+24hx-h}{h} = \frac{h(12h+24x-1)}{h} = \underline{12h+24x-1}$$

$$f'(x) = 3(4x-3)+(3x+2)\cdot 4$$

$$= 12x-9+12x+8 = \underline{\underline{24x-1}}$$

oppgave 4 Se tabell 7.1

a) $f(x) = x^2 + x + 4 - \pi$

$\underline{\underline{f'(x) = 2x + 1}}$

b) $f(x) = x^8 - x^5 + x^3 + 3$

$\underline{\underline{f'(x) = 8x^7 - 5x^4 + 3x^2}}$

c) $f(v) = v^{17} + v^{14} - v^{10} - 5v$

$\underline{\underline{f'(v) = 17v^{16} + 14v^{13} - 10v^9 - 5}}$

* d) $f(x) = 5x^6 + 2x^3 - 3x$

$\underline{\underline{f'(x) = 30x^5 + 6x^2 - 3}}$

e) $f(x) = \frac{5}{6}x^{12} - \frac{2}{3}x^3$

$f'(x) = \frac{12 \cdot 5}{6}x^{11} - \frac{3 \cdot 2}{3}x^2$

$= \underline{\underline{10x^{11} - 2x^2}}$

* f) $f(t) = \frac{t^7}{7} + \frac{t^4}{2} - \frac{t}{3} + \frac{7}{5}$

$f'(t) = \frac{7t^6}{7} + \frac{4t^3}{2} - \frac{1}{3} = \underline{\underline{t^6 + 2t^3 - \frac{1}{3}}}$

Oppgave 5

Finn tangentens likning i det angitte punktet

a) $y = f(x) = 3x^2 - 2x + 1$ $(1,2)$

Tangentens likning er gitt ved følgende formel:

$y = f'(a)(x-a) + f(a)$

gitt at: $x = 1$ gir $f(1) = 2$ [dvs: $a = 1$ og $f(a) = 2$]

$f'(x) = 6x - 2$ $f'(1) = 6 - 2 = 4$

Vi får følgende likning:

$y = 4(x-1) + 2$

$= \underline{\underline{4x - 2}}$

b) $y = f(x) = 4x^4 + 5x^3 - 2$ $(-1, -3)$

gitt at $x = -1$ gir $f(x) = -3$ [dvs: $a = -1$ og $f(a) = -3$]

$f'(x) = 16x^3 + 15x^2$ $f'(a) = f'(-1) = -16 + 15 = -1$

Vi får følgende likning:

$y = -1(x - (-1)) - 3$

$= -x - 1 - 3 = \underline{\underline{-x - 4}}$

oppgave 7 Se tabell 7.2

a) * $f(x) = (x^2-2)(x+6)$

$f'(x) = 2x(x+6) + (x^2-2)\cdot 1$

$= 2x^2 + 12x + x^2 - 2 = \underline{\underline{3x^2 + 12x - 2}}$

b) $f(x) = (3x^2+x-3)(x^2-4x+1)$

$f'(x) = (6x+1)(x^2-4x+1) + (3x^2+x-3)(2x-4)$

$= 6x^3 - 24x^2 + 6x + x^2 - 4x + 1 + (6x^3 - 12x^2 + 2x^2 - 4x - 6x + 12)$

$= \underline{\underline{12x^3 - 33x^2 - 8x + 13}}$

c) $f(x) = (2x^3+5)(x^2-2)(5x+1)$

$= (2x^3+5)(5x^3+x^2-10x-2)$

$f'(x) = 6x^2(5x^3+x^2-10x-2) + (2x^3+5)(15x^2+2x-10)$

$= 30x^5 + 6x^4 - 60x^3 - 12x^2 + (30x^5 + 4x^4 - 20x^3 + 75x^2 + 10x - 50)$

$= \underline{\underline{60x^5 + 10x^4 - 80x^3 + 63x^2 + 10x - 50}}$

d) $f(t) = (t+3)(t^2+1)(t^3-1)$

$= (t^3+t+3t^2+3)(t^3-1)$

$f'(t) = (3t^2+6t+1)(t^3-1) + (t^3+3t^2+t+3)3t^2$

$= 3t^5 + 6t^4 + t^3 - 3t^2 - 6t - 1 + 3t^5 + 9t^4 + 3t^3 + 9t^2$

$= \underline{\underline{6t^5 + 15t^4 + 4t^3 + 6t^2 - 6t - 1}}$

e) $f(x) = 7x^3 + \frac{3}{x^2}$

$f'(x) = 21x^2 + \frac{0\cdot x^2 - 3\cdot 2x}{x^4}$

$= \underline{\underline{21x^2 - \frac{6}{x^3}}}$

Oppgave 7

f) $f(x) = \frac{2}{x^3} + \frac{3}{x^2} - \frac{6}{x}$

$$f'(x) = \frac{(0 \cdot x^3 - 2 \cdot 3x^2)}{x^6} + \frac{(0 \cdot x^2 - 3 \cdot 2x)}{x^4} - \frac{(0 \cdot x - 6 \cdot 1)}{x^2}$$

$$= -\frac{6x^2}{x^6} - \frac{6x}{x^4} + \frac{6}{x^2}$$

$$= \underline{\underline{-\frac{6}{x^4} - \frac{6}{x^3} + \frac{6}{x^2}}}$$

Oppgave 8

a) $f(x) = (3x^2 + 5)^4$

$u = g(x) = 3x^2 + 5$

$f(u) = u^4$

$\frac{du}{dx} = g'(x) = 6x \qquad \frac{df}{du} = 4u^3$

Dette gir:

$$f'(x) = f'(u)\, g'(x) = \frac{df}{du}\frac{du}{dx} = 4(3x^2+5)^3 \cdot 6x$$

$$= \underline{\underline{24x(3x^2+5)^3}}$$

$\uparrow$

Kjerneregelen!

b) * $f(x) = (-4x^3 + 5x^2 + 3)^7$

$u = g(x) = -4x^3 + 5x^2 + 3$

$f(u) = u^7$

$\frac{du}{dx} = g'(x) = -12x^2 + 10x \qquad \frac{df}{du} = 7u^6$

Dette gir:

$$f'(x) = f'(u)g'(x) = \frac{df}{du}\frac{du}{dx} = 7(-4x^3+5x^2+3)^6(-12x^2+10x)$$

$$= \underline{\underline{-14x(6x-5)(-4x^3+5x^2+3)^6}}$$

oppgave 8

d) $f(x) = (7x^3+2)^3(6x^2-1)^4$

$f(x) = m(x) \cdot n(x)$

$m(x) = (7x^3+2)^3$ $u = g(x) = 7x^3+2$ $f(u) = u^3$ $g'(x) = 21x^2$ $f'(u) = 3u$

$m'(x) = f'(u) \cdot g'(x)$

$= 3u^2 \cdot 21x^2 = 3(7x^3+2)^2 \cdot 21x^2 = 63x^2(7x^3+2)^2$

$n(x) = (6x^2-1)^4$ $u = g(x) = 6x^2-1$ $f(u) = u^4$ $g'(x) = 12x$ $f'(u) = 4u$

$n'(x) = f'(u) \cdot g'(x)$

$= 4u^3 \cdot 12x = 48x(6x^2-1)^3$

$f'(x) = m'(x) \cdot n(x) + m(x) \cdot n'(x)$ Se tabell 7.2

$= 63x^2(7x^3+2)^2(6x^2-1)^4 + (7x^3+2)^3 48x(6x^2-1)^3$

$= 3x(7x^3+2)^2(6x^2-1)^3(21x(6x^2-1) + 16(7x^3+2))$

$= 3x(7x^3+2)^2(6x^2-1)^3(126x^3 - 21x + 112x^3 + 32)$

$= \underline{\underline{3x(7x^3+2)^2(6x^2-1)^3(238x^3 - 21x + 32)}}$

oppgave 8

c) $f(t) = (2t^5 - 3t + 4)^{-3}$

$u = g(t) = 2t^5 - 3t + 4$ $f(u) = u^{-3}$

$\frac{du}{dt} = g'(t) = 10t^4 - 3$ $\frac{df}{du} = f'(u) = -3u^{-4}$

Dette gir:

$f'(t) = f'(u)g'(t) = \underline{\underline{-3(2t^5-3t+4)^{-4}(10t^4-3)}}$

Oppgave 9

a) $f(x) = \sqrt[7]{x^3} = x^{\frac{3}{7}}$

$f'(x) = \frac{3}{7} x^{\frac{3}{7}-1} = \frac{3}{7} x^{-\frac{4}{7}} = \frac{3}{7} \sqrt[7]{x^{-4}} = \underline{\underline{\frac{3}{7\sqrt[7]{x^4}}}}$

b) $f(x) = \frac{1}{\sqrt[5]{x^2}} = \sqrt[5]{x^{-2}} = x^{-\frac{2}{5}}$

$f'(x) = -\frac{2}{5} x^{-\frac{2}{5}-1} = -\frac{2}{5} x^{-\frac{7}{5}} = \frac{-2}{5\sqrt[5]{x^7}} = \frac{-2}{5\sqrt[5]{x^5}\sqrt[5]{x^2}}$

$= \underline{\underline{\frac{-2}{5x\sqrt[5]{x^2}}}}$

Siden $\sqrt[5]{x^5} = x^{\frac{5}{5}} = x^1 = x$

c) $f(z) = z^3 \sqrt[4]{\sqrt{z}} = z^3 (\sqrt{z})^{\frac{1}{4}} = z^3 (z^{\frac{1}{2}})^{\frac{1}{4}} = z^3 z^{\frac{1}{8}}$

$f'(z) = 3z^2 z^{\frac{1}{8}} + z^3 \frac{1}{8} z^{-\frac{7}{8}}$ Se tabell 7.2

$= 3z^{\frac{17}{8}} + \frac{1}{8} z^{\frac{17}{8}} = \frac{25}{8} z^{\frac{17}{8}} = \frac{25}{8} \sqrt[8]{z^{17}}$

$= \frac{25}{8} \sqrt[8]{z^{16}} \sqrt[8]{z}$

Siden $\sqrt[8]{x^{16}} = x^{\frac{16}{8}} = x^2$

$= \underline{\underline{\frac{25}{8} z^2 \sqrt[8]{z}}}$

eller

$z^3 \cdot z^{\frac{1}{8}} = z^{3+\frac{1}{8}} = z^{\frac{24}{8}+\frac{1}{8}} = z^{\frac{25}{8}}$

$\downarrow$

osv

oppgave 10

a) $f(x) = \frac{1}{\cos x}$

$$f'(x) = \frac{0 \cdot \cos x - 1(-\sin x)}{\cos^2 x} = \underline{\underline{\frac{\sin x}{\cos^2 x}}}$$

b) $f(x) = \sin(3x) \cdot \sin x$

$$f'(x) = \underline{\underline{3\cos(3x) \cdot \sin x + \sin(3x)\cos x}}$$

$g_1(x) = u = 3x$ $\quad$ $f(u) = \sin u$

$g'(x) = \frac{du}{dx} = 3$ $\quad$ $f'(u) = \cos u$

$$m'(x) = f'(u)\,g'(x)$$
$$= \cos u \cdot 3$$
$$= 3\cos(3x)$$

c) * $f(t) = \cos^3\left(\frac{t}{5}\right) = \left(\cos \frac{t}{5}\right)^3$

Vi benytter kjerneregelen to ganger!

$z = k(t) = \frac{t}{5}$ $\qquad$ $\frac{dz}{dt} = k'(t) = \frac{1}{5}$

$u = g(z) = \cos z$ $\qquad$ $\frac{du}{dz} = g'(z) = -\sin z$

$f(u) = u^3$ $\qquad$ $\frac{df}{du} = f'(u) = 3u^2$

$$f'(t) = f'(u)\,g'(z)\,k'(t) = \frac{df}{du}\frac{du}{dz}\frac{dz}{dt}$$
$$= 3u^2 \cdot -\sin z \cdot \frac{1}{5}$$
$$= -\frac{1}{5}\, 3\cos^2 z \sin z$$
$$= \underline{\underline{-\frac{3}{5}\cos^2\left(\frac{t}{5}\right)\sin\left(\frac{t}{5}\right)}}$$

oppgave 11

a) $f(x) = \sin^{-1}(7x)$

$u = g(x) = 7x \qquad \frac{du}{dx} = g'(x) = 7$

$f(u) = \sin^{-1}(u) \qquad \frac{df}{du} = \frac{1}{\sqrt{1-u^2}}$

$f'(x) = f'(u)\,g'(x)$

$= \frac{1}{\sqrt{1-u^2}} \cdot 7 = \frac{7}{\sqrt{1-(7x)^2}} = \underline{\underline{\frac{7}{\sqrt{1-49x^2}}}}$

b) $h(t) = t^3 \sin^{-1}(t)$

$h'(t) = 3t^2 \sin^{-1}(t) + t^3 \frac{1}{\sqrt{1-t^2}}$

$= 3t^2 \sin^{-1}(t) + \frac{t^3}{\sqrt{1-t^2}}$

c) $f(x) = \cos^{-1}(\sin x)$

$u = g(x) = \sin x \qquad \frac{du}{dx} = g'(x) = \cos x$

$f(u) = \cos^{-1}(u) \qquad \frac{df}{du} = \frac{-1}{\sqrt{1-u^2}}$

$f'(x) = f'(u)\,g'(x) = \frac{-1}{\sqrt{1-u^2}} \cos x = \frac{-\cos x}{\sqrt{1-\sin^2 x}} = \frac{-\cos x}{\sqrt{\cos^2 x}}$

$= \underline{\underline{-1}}$

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

oppgave 12

a) $f(x) = \ln(-5x)$

$u = g(x) = -5x$ $\quad \frac{du}{dx} = g'(x) = -5$

$f(u) = \ln(u)$ $\quad \frac{df}{du} = f'(u) = \frac{1}{u}$

$f'(x) = f'(u) g(x) = \frac{1}{u}(-5) = \frac{1}{-5x}(-5) = \underline{\underline{\frac{1}{x}}}$

b) $f(z) = \ln \sqrt[6]{z^5}$

$u = g(z) = \sqrt[6]{z^5} = z^{\frac{5}{6}}$ $\quad \frac{du}{dz} = g'(z) = \frac{5}{6} z^{-\frac{1}{6}}$

$f(u) = \ln u$ $\quad \frac{df}{du} = f'(u) = \frac{1}{u}$

$f'(z) = f'(u) g'(z) = \frac{1}{u} \frac{5}{6} z^{-\frac{1}{6}} = \frac{1}{z^{5/6}} \frac{5}{6} z^{-\frac{1}{6}}$

$= \frac{5}{6 \sqrt[6]{z^5} \cdot \sqrt[6]{z}} = \frac{5}{6 \sqrt[6]{z^6}} = \underline{\underline{\frac{5}{6z}}}$

Oppgave 13

a) $$y = f(x) = \frac{(3x+1)^3(x^2+3)}{(4x^3+7)^5}$$

$$\ln f(x) = 3\ln(3x+1) + \ln(x^2+3) - 5\ln(4x^3+7)$$

$$\frac{1}{f(x)} f'(x) = 0\cdot\ln(3x+1) + 3\,\frac{1}{3x+1}\cdot 3 + \frac{1}{x^2+3}\cdot 2x - (0\cdot\ln(4x^3+7)\cdot 5\cdot\frac{1}{4x^3+7}\,12x^2$$

$$= \frac{9}{3x+1} + \frac{2x}{x^2+3} - \frac{60x^2}{4x^3+7}$$

$$\underline{\underline{f'(x) = f(x)\left[\frac{9}{3x+1} + \frac{2x}{x^2+3} - \frac{60x^2}{4x^3+7}\right]}}$$

Oppgave 14

a) $f(x) = e^{5x^4+6x-1}$

$\ln f(x) = 5x^4 + 6x - 1 \ \ln e$ | tar ln

$\frac{1}{f(x)} f'(x) = 20x^3 + 6$ | deriverer

$f'(x) = f(x)\,(20x^3+6)$

$= \underline{\underline{(20x^3+6)\,e^{5x^4+6x-1}}}$

b) $f(x) = \frac{1}{\sqrt[7]{e^{4x}}} = e^{-\frac{4x}{7}}$

$\ln f(x) = -\frac{4x}{7} \ln e$ | tar ln

$\frac{1}{f(x)} f'(x) = -\frac{4}{7}$ | deriverer

$f'(x) = f(x) \cdot -\frac{4}{7}$

$= \underline{\underline{-\frac{4}{7} \cdot \frac{1}{\sqrt[7]{e^{4x}}}}}$

Kan også løses ved å benytte kjerneregel, se neste side!

oppgave 14

a) $f(x) = e^{5x^4+6x-1}$

$g(x) = u = 5x^4+6x-1$

$g'(x) = \frac{du}{dx} = 20x^3+6$

$f(u) = e^u \qquad f'(u) = \frac{df}{du} = e^u$

$f'(x) = \frac{df}{du}\frac{du}{dx} = f'(u)\cdot g'(x) = e^u(20x^3+6)$

$= \underline{\underline{(20x^3+6)e^{5x^4+6x-1}}}$

b) $f(x) = \frac{1}{\sqrt[7]{e^{4x}}} = \left(e^{-4x}\right)^{\frac{1}{7}} = e^{-\frac{4}{7}x}$

$u = g(x) = -\frac{4}{7}x$

$\frac{du}{dx} = g'(x) = -\frac{4}{7}$

$f(u) = e^u \qquad f'(u) = \frac{df}{du} = e^u$

$f'(x) = f'(u)\,g'(x) = \frac{df}{du}\frac{du}{dx} = e^u \cdot -\frac{4}{7} = -\frac{4}{7}e^{-\frac{4}{7}x}$

$= \underline{\underline{\frac{-4}{7\sqrt[7]{e^{4x}}}}}$

Oppgave 15

a) $f(x) = -5x^3 + 4x^2 + 3$

$f'(x) = -15x^2 + 8x$

$f''(x) = -30x + 8$

$f'''(x) = -30$

b) $f(x) = x^3 \cdot \ln x$

$f'(x) = 3x^2 \cdot \ln x + x^3 \cdot \frac{1}{x}$

$= 3x^2 \ln x + x^2$

$f''(x) = 6x \cdot \ln x + 3x^2 \cdot \frac{1}{x} + 2x$

$= 6x \ln x + 3x + 2x$

$= 6x \ln x + 5x$

$f'''(x) = 6 \ln x + 6x \cdot \frac{1}{x} + 5$

$= 6 \ln x + 6 + 5$

$= 6 \ln x + 11$